Variable structure tracking control for flexible spacecraft

Dong Ye and Zhaowei Sun
Research Center of Satellite Technology, Harbin Institute of Technology, Harbin, China

Abstract
Purpose – This paper aims to present a three-axis attitude tracking control law to solve the attitude maneuver of a flexible satellite in the presence of parameter uncertainties and external disturbance.
Design/methodology/approach – Based on the relative dynamic equation where the relative attitude is described by quaternion, a robust control law composed of a proportional derivative (PD) part plus a signum function is designed and only requires the measurement of attitude and angular velocity. Furthermore, the stability analysis of the proposed control law is given through a two-step proof technique.
Findings – Numerical simulation results demonstrate that fine convergence of the attitude and angular velocity error and low-level vibration of flexible appendages are obtained by the proposed controllers.
Practical implications – The controller with the structure of a PD term plus a switching function about a sliding variable has low computational complexity and does not need to measure the modal variables of elastic appendages, so it can be used in orbit without modification.
Originality/value – The globally asymptotic stability of the controller in the presence of model uncertainties and external disturbances is proven rigorously through a two-step proof technique.

Keywords Attitude tracking control, Flexible spacecraft, Globally asymptotic stability, Lyapunov methods

Paper type Research paper

Nomenclature

q = unit quaternion
q_o = the scalar part of a quaternion
q_e = the vector part of a quaternion
ω = angular velocity of spacecraft (rad/s)
r = the subscript represents the desired motion
e = the subscript represents tracking error
$\mathcal{J}$ = inertia matrix of the spacecraft (kg·m^2)
δ = the coupling matrix between the central rigid body and the flexible attachments (kg$^{1/2}$·m)
η = the modal coordinate vector
τ = the control input (N·m)
d = the external disturbance torque (N·m)
ω_n = the natural frequencies of the flexible attachment (rad/s)
ω_e = attitude angular velocity tracking error (rad/s)
ω_r = the desired angular velocity (rad/s)
ω_b = angular velocity of spacecraft (rad/s)
K = the stiffness matrix
C = the damping matrix
ϑ = $(\eta^T \quad (\dot{\eta}+\delta\omega_b)^T)^T$
$\mathcal{J}_{mb}$ = $\mathcal{J} - \delta^T\delta(\text{kg}^{1/2} \times \text{m})$
L = $(\delta^T K \quad \delta^T C)$
ς = $(\|q_e\| \quad \|\omega_e\|)^T$
M = $\delta^T C\delta$
A = $\begin{pmatrix} 0 & I \\ -K & -C \end{pmatrix}$
B = $\begin{pmatrix} -\delta \\ C\delta \end{pmatrix}$
X = a Hermitian matrix
$x(t)$ = a general piecewise continuous function
L_p = L_p space
L_∞ = L_∞ space
L_2 = L_2 space
P,Q = a positive definite matrix
ξ = $(\|q_e\| \quad \|\omega_e\| \quad \|\vartheta\|)^T$
ξ = the associated damping of the flexible attachment
s = the sliding mode surface
C_{bd} = the rotation matrix from the desired body frame to the body-fixed frame
k_p = the proportional value in controller
k_d = the derivative value in controller
λ = the gain for sign term in controller
V_1, V_2 = Lyapunov or Lyapunov-like function
σ = the singular value of matrix f, K_1, K_2, K_3, Θ_1, U, V, W, g, K_4 and Θ_2= matrices

Definitions, Acronyms and Abbreviations

PD = proportional derivative

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Introduction

Modern spacecraft often use large, complex and lightweight structures such as solar arrays and antennas to achieve increased functionality at a reduced launch cost and provide agile slewing capabilities (Wu *et al.*, 2013). Unfortunately, making a mechanical system lightweight usually means that these space structures are extremely flexible and have low-frequency fundamental vibration modes. These modes might be excited at high accelerations in a variety of tasks such as slewing and pointing maneuvers. This vibration can cause a variety of problems including positioning errors, slow overall move times (if vibration must naturally damp out) and system damage. To get satisfactory control performance in the case of these detrimental factors is a challenging task for the spacecraft designers. Various approaches have been proposed to deal with such a problem.

The tracking control problem can be treated as the well-known rigid body control issue when the displacement of elastic appendages is not taken into account. The representations of attitude error between the body frame and the desired coordinate frame are summarized in Wen and Kreutz-Delgado (1991), and the proportional derivative (PD) and other modified PD controllers were designed for the case without disturbance in which the robust controller for model uncertainties was emphasized. Based on the relative attitude kinematics and dynamics equations using modified Rodrigues parameters to represent the relative attitude (Xing and Parvez, 2001), G. Q. Xing presented state tracking controllers for rigid body maneuver (Xing, 1999). Crassidis *et al.* (2000) designed a variable structure feedback controller which provides global asymptotic tracking of spacecraft maneuvers in the presence of either external control torques or reaction wheel internal torques using the multiplicative error quaternion definition to denote the reference trajectory tracking errors. When considering the torque saturation in the practical attitude control problem, Boskovic recently proposed asymptotically stable control laws for robust attitude controller that takes into account control saturation explicitly and achieves effective compensation of external disturbances and dynamic model uncertainty (Boskovic *et al.*, 2001, 2004).

The design of the controller becomes much more complicated when the displacement of elastic appendages is considered. As the sliding mode control is an effective approach to deal with parametric uncertainties and external disturbances for dynamic systems because of its simplicity and effectiveness, as well as its robustness (Young *et al.*, 1999), it was applied in attitude controller design for a flexible spacecraft in Iyer and Singh (1988, 1989). However, it is necessary to obtain the bound of model variables to ensure stability. The adaptive controller was designed for the condition in which the inertia matrix is uncertain and the gravitational torque is state-dependent. However, the states which cannot be measured by existing sensors were used in the control law. Furthermore, Gennaro (2003) proposed a dynamic controller that ensures the tracking of a desired attitude characterized by bounded velocity and acceleration without the spacecraft angular velocity in presence of disturbances with bounded dynamics.

In this research, the attitude tracking problem for a flexible spacecraft with a general desired trajectory is studied. The main contribution is that the presented variable structure control law can render the attitude and angular velocity tracking errors globally asymptotically stable rather than ultimately bounded in the face of model uncertainties and unexpected disturbances. In addition, there is no prior assumption on the bound of the model uncertainties and unexpected disturbance.

The remainder of this paper is arranged as follows. Preliminaries for attitude kinematics and dynamics are given in the following section. In Section 3, a Lyapunov-based PD plus variable structure tracking control algorithm is proposed. The stability analyses are then provided for the given controllers. Sections 4 presents numerical simulation results, and Section 5 gives the conclusion, followed by references.

Attitude kinematics and dynamics

The unit quaternion $q(t) = \{q_o(t), q_e(t)\}$ describes the orientation of the body-fixed frame with respect to the desired reference frame, which is defined as:

$$q_0 = \cos(\Phi/2), \quad q_e = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \upsilon \sin(\Phi/2) \tag{1}$$

This equation denotes the result of a virtual rotation by eigenaxis rotation angle Φ about a virtual unit axis vector υ (known as eigenaxis) and is subjected to the constraint:

$$q_e^T q_e + q_0^2 = 1 \tag{2}$$

The kinematic equation for quaternion is expressed as:

$$\begin{aligned} \dot{q}_e &= \frac{1}{2}(q_e^{\times}\omega + q_0\omega) \\ \dot{q}_0 &= -\frac{1}{2}q_e^T\omega \end{aligned} \tag{3}$$

The notation $\zeta^{\times}$, $\forall\zeta = [\zeta_1, \zeta_2, \zeta_3]^T$, denotes the following skew-symmetric matrix:

$$\zeta^{\times} \triangleq \begin{bmatrix} 0 & -\zeta_3 & \zeta_2 \\ \zeta_3 & 0 & -\zeta_1 \\ -\zeta_2 & \zeta_1 & 0 \end{bmatrix} \tag{4}$$

Without loss of generality, we just consider the spacecraft with one flexible appendage. The equation governing the flexible spacecraft is expressed as (Jin and Sun, 2010):

$$\begin{aligned} \mathcal{J}\dot{\omega}_b + \delta^T\ddot{\eta} &= -\omega_b^{\times}(\mathcal{J}\omega_b + \delta^T\dot{\eta}) + \tau + d \\ \ddot{\eta} + C\dot{\eta} + K\eta &= -\delta\,\dot{\omega}_b \end{aligned} \tag{5}$$

where $\mathcal{J} = \mathcal{J}^T$ is the total inertia matrix of the spacecraft, ω_b is angular velocity of spacecraft with respect to inertial frame expressed in body-fixed frame, δ is the coupling matrix between the central rigid body and the flexible attachments, namely, the matrix which describes how the flexile dynamics influences the rigid dynamics, and vice versa, η is the modal coordinate vector relative to the main body, τ denotes the control input acting on the main body of the spacecraft, d represents the external disturbance torque and K and C

denote the stiffness and damping matrices, respectively, which are defined as:

$$\begin{aligned} C &= \mathrm{diag}(2\xi_i\,\omega_{ni}, i = 1, 2, \cdots, N) \\ K &= \mathrm{diag}(\omega_{ni}^2, i = 1, 2, \cdots, N) \end{aligned} \tag{6}$$

In the present model, N elastic modes are taken into consideration, with ω_{ni}, the ist natural frequencies, and ξ_i, the ist associated dampings.

From equation (5), it is possible to obtain the dynamics of the flexible spacecraft:

$$\begin{aligned} \mathcal{J}_{mb}\dot{\omega}_b &= -\omega_b^\times(\mathcal{J}_{mb}\omega_b + H\vartheta) + L\vartheta - M\omega_b + \tau + d \\ \dot{\vartheta} &= A\vartheta + B\omega_b \end{aligned} \tag{7}$$

where $\vartheta = (\eta^T \quad (\dot{\eta}+\delta\omega_b)^T)^T$. The matrices $\mathcal{J}_{mb}$, H, L, M, A and B are given as:

$$\begin{aligned} &\mathcal{J}_{mb} = \mathcal{J} - \delta^T\delta,\ H = \begin{pmatrix} 0 & \delta^T \end{pmatrix} \\ &L = \begin{pmatrix} \delta^T K & \delta^T C \end{pmatrix},\ M = M^T = \delta^T C\delta \\ &A = \begin{pmatrix} 0 & I \\ -K & -C \end{pmatrix},\ B = \begin{pmatrix} -\delta \\ C\delta \end{pmatrix} \end{aligned}$$

Clearly, A is a Hurwitz matrix.

Attitude angular velocity tracking error ω_e can be described as:

$$\omega_e = \omega_b - C_{bd}\,\omega_d \tag{8}$$

where, C_{bd} is the rotation matrix from the desired body frame to the body-fixed frame, and ω_r represents the desired angular velocity.

Substituting equation (8) into equation (7), the relative dynamic equation of flexible spacecraft can be expressed as:

$$\begin{aligned} \mathcal{J}_{mb}\,\dot{\omega}_e = &-\omega_e^\times\mathcal{J}_{mb}\omega_e - \omega_e^\times\mathcal{J}_{mb}C_{br}\omega_r - (C_{br}\omega_r)^\times\mathcal{J}_{mb}\omega_e \\ &- \omega_e^\times H\vartheta - M\omega_e + \mathcal{J}_{mb}\omega_e^\times C_{br}\omega_r \\ &- (C_{br}\omega_r)^\times\mathcal{J}_{mb}C_{br}\omega_r - (C_{br}\omega_r)^\times H\vartheta + L\vartheta \\ &- MC_{br}\omega_r - \mathcal{J}_{mb}C_{br}\dot{\omega}_r + \tau + d \\ \dot{\vartheta} = &\ A\vartheta + B\omega_e + BC_{br}\omega_r \end{aligned} \tag{9}$$

Variable structure tracking controller design

In this section, the main results of this paper are presented. The control goal is that from any initial state, the tracking system error (including attitude tracking error and angular velocity tracking error) and the mode variables of the flexible appendage can be controlled to a closed set containing zero, and also the attitude and angular velocity tracking error converge to zero, that is, $\lim_{t\to\infty} q_e(t) = 0$ and $\lim_{t\to\infty} \omega_e(t) = 0$.

Before designing the controller, two lemmas are presented here first, which will be used in the following stability analysis.

Lemma 1: Schur complement (Boyd *et al.*, 1994)

Let A be a Hermitian matrix ($X = X^*$) partitioned as:

$$X = \begin{pmatrix} X_{11} & X_{12} \\ X_{12}^* & X_{22} \end{pmatrix} \tag{10}$$

The necessary and sufficient conditions for positive definite of matrix X are one of the following conditions:

1. $X_{11} - X_{12}X_{22}^{-1}X_{12}^* > 0$, where $X_{22} > 0$; and
2. $X_{22} - X_{12}^*X_{11}^{-1}X_{12} > 0$, where $X_{11} > 0$.

Lemma 2: Barbalat lemma (Slotine and Li, 1991)

If $f(t)$, $\dot{f}(t) \in L_\infty$ and $f(t) \in L_p$ for some $p \in [1, \infty)$, then $\lim_{t\to\infty} f(t) = 0$, where a general piecewise continuous function $x(t) \in L_p$ means $(\int_0^\infty \|x(t)\|^p dt)^{1/p} < \infty$ and, especially, $x(t) \in L_\infty$ means $\sup_{t\geq 0}\|x(t)\| < \infty$.

Consider the following controller:

$$\tau = -k_p q_e - k_d\omega_e - f(s) \tag{11}$$

where k_p and k_d are both positive constants, and $f(s)$ is of the form:

$$f_i(s) = \lambda_i\,\mathrm{sgn}\,(s_i),\ i = 1, 2, 3 \tag{12}$$

with $s_i = \omega_{ei} + c_i q_{ei}$, where λ_i and c_i are both positive constants. The sign function sgn (u) is defined as:

$$\mathrm{sgn}\,(u) = \begin{cases} 1, & u \geq 0 \\ -1, & u < 0 \end{cases} \tag{13}$$

Theorem 1

For suitable k_p, k_d, λ and sufficiently small positive c, the system (3) and (5) with variable structure tracking controller (11) about the desired attitude states $q_e = 0$ and $\omega_e = 0$ are asymptotically stable for any initial state $(q_e(0), \omega_e(0)) \in \Re^6$.

Proof

The procedure of the proof is to choose a proper Lyapunov function first, with which the ultimate boundedness of the tracking errors and modal variables is achieved in finite time, and then to select another Lyapunov function, with which and the preceding ultimate boundedness result, the asymptotic stability of the tracking errors q_e and ω_e are guaranteed. That is, the proof of the theorem includes the following two consecutive steps:

1. *Step 1*: The tracking errors variables q_e and ω_e are bounded under the effect of the controller.
2. *Step 2*: The tracking errors q_e and ω_e are asymptotically stable with bounded tracking errors and modal variables.

Proof of Step 1

Consider the following scalar function:

$$\begin{aligned} V_1 = &(k_p + ck_d)\left((1 - q_0)^2 + q_e^T q_e\right) + \frac{1}{2}\omega_e^T\mathcal{J}_{mb}\omega_e \\ &+ cq_e^T\mathcal{J}_{mb}\omega_e + \frac{1}{2}\vartheta^T P\vartheta \end{aligned} \tag{14}$$

where P is a positive definite matrix, which is the solution of the Lyapunov equation $A^T P + PA = -Q$ with a positive definite matrix Q. Suppose $q_0 \geq 0$. The scalar function V_1 such as energy function of the flexible spacecraft, which is used to achieve the stability property via Lyapunov method (Khalil and Grizzle, 2001), can be bounded as:

$$\frac{1}{2}\xi^T\Xi_1\xi \leq V_1 \leq \frac{1}{2}\,\xi^T\Xi_2\xi \tag{15}$$

where $\xi = (\|q_e\| \quad \|\omega_e\| \quad \|\vartheta\|)^T$.

$$\Xi_1 = \Xi_1^{\mathrm{T}} = \begin{pmatrix} 2(k_P + ck_d) & -c\sigma_{\max}(\mathcal{J}_{\mathrm{mb}}) & 0 \\ -c\sigma_{\max}(\mathcal{J}_{\mathrm{mb}}) & \sigma_{\min}(\mathcal{J}_{\mathrm{mb}}) & 0 \\ 0 & 0 & \sigma_{\min}(P) \end{pmatrix},$$

$$\Xi_2 = \Xi_2^{\mathrm{T}} = \begin{pmatrix} 4(k_P + ck_d) & c\sigma_{\max}(\mathcal{J}_{\mathrm{mb}}) & 0 \\ c\sigma_{\max}(\mathcal{J}_{\mathrm{mb}}) & \sigma_{\max}(\mathcal{J}_{\mathrm{mb}}) & 0 \\ 0 & 0 & \sigma_{\max}(P) \end{pmatrix},$$

where $\sigma_{\min}(\cdot)$ and $\sigma_{\max}(\cdot)$ denote the minimum and maximum singular values of a positive matrix, respectively. According to Lemma 1, V_1 is positive definite when c is sufficiently small.

The time derivative of V_1 along the solution trajectory can be deduced as:

$$\begin{aligned}\dot{V}_1 = {} & (k_p + ck_d)q_{\mathrm{e}}^{\mathrm{T}}\omega_{\mathrm{e}} + (\omega_{\mathrm{e}} + cq_{\mathrm{e}})^{\mathrm{T}}(\tau + f) + q_{\mathrm{e}}^{\mathrm{T}}K_1\omega_{\mathrm{e}} \\ & + q_{\mathrm{e}}^{\mathrm{T}}K_2\vartheta + \omega_{\mathrm{e}}^{\mathrm{T}}K_3\vartheta - \omega_{\mathrm{e}}^{\mathrm{T}}(M + (C_{\mathrm{br}}\omega_{\mathrm{r}})^{\times}\mathcal{J}_{\mathrm{mb}} \\ & + \mathcal{J}_{\mathrm{mb}}(C_{\mathrm{br}}\omega_{\mathrm{r}})^{\times})\omega_{\mathrm{e}} + \frac{1}{2}c\omega_{\mathrm{e}}^{\mathrm{T}}(q_{\mathrm{e}0}I + q_{\mathrm{e}}^{\times})\mathcal{J}_{\mathrm{mb}}\omega_{\mathrm{e}} \\ & - \frac{1}{2}\vartheta^{\mathrm{T}}Q\vartheta + \vartheta^{\mathrm{T}}PBC_{\mathrm{br}}\omega_{\mathrm{r}} \end{aligned} \tag{16}$$

where:

$$\begin{aligned} f &= d - (C_{\mathrm{br}}\omega_{\mathrm{r}})^{\times}\mathcal{J}_{\mathrm{mb}}C_{\mathrm{br}}\omega_{\mathrm{r}} - \mathcal{J}_{\mathrm{mb}}C_{\mathrm{br}}\dot{\omega}_{\mathrm{r}} - MC_{\mathrm{br}}\omega_{\mathrm{r}} \\ K_1 &= c(\mathcal{J}_{\mathrm{mb}}C_{\mathrm{br}}\omega_{\mathrm{r}})^{\times} - c(C_{\mathrm{br}}\omega_{\mathrm{r}})^{\times}\mathcal{J}_{\mathrm{mb}} - c\mathcal{J}_{\mathrm{mb}}(C_{\mathrm{br}}\omega_{\mathrm{r}})^{\times} - cM \\ K_2 &= cL\vartheta - c(C_{\mathrm{br}}\omega_{\mathrm{r}})^{\times}H \\ K_3 &= L + B^{\mathrm{T}}P - (C_{\mathrm{br}}\omega_{\mathrm{r}})^{\times}H + cq_{\mathrm{e}}^{\times}H \end{aligned}$$

Substituting the control law (11) into (16) yields:

$$\begin{aligned}\dot{V}_1 \le {} & -\xi^{\mathrm{T}}\Theta_1\xi - \sum_{i=1}^{3}\left(\lambda - \sup_{t\in[0,\infty)}|f_i(t)|\right)|s_i| \\ & + \sigma_{\max}(PB)\sup_{t\in[0,\infty)}\|\omega_{\mathrm{r}}\|\,\|\xi\| \end{aligned} \tag{17}$$

where Θ_1 is given as:

$$\Theta_1 = \begin{pmatrix} U & V \\ V^{\mathrm{T}} & W \end{pmatrix}$$

where:

$$U = \begin{pmatrix} ck_p & -\frac{1}{2}\sigma_{\max}(K_1) \\ -\frac{1}{2}\sigma_{\max}(K_1) & k_d + \sigma_{\min}(M) - \frac{1}{2}c\sigma_{\max}(\mathcal{J}_{\mathrm{mb}}) \end{pmatrix}$$

$$V = \begin{pmatrix} -\frac{1}{2}\sigma_{\max}(K_2) \\ -\frac{1}{2}\sigma_{\max}(K_3) \end{pmatrix}, \quad W = \frac{1}{2}\sigma_{\min}(Q)$$

According to Lemma 1, we know that there exists appropriate controller parameters c, k_P and k_D such that Θ_1 is positive definite, and if $\lambda \ge \sup_{t\in[0,\infty)}|f_i(t)|$, $i = 1, 2, 3$, then $\dot{V}_1$ satisfies the following inequality:

$$\begin{aligned}\dot{V}_1 &\le -\sigma_{\min}(\Theta_1)\|\xi\|^2 + \sigma_{\max}(PB)\sup_{t\in[0,\infty)}\|\omega_{\mathrm{r}}\|\,\|\xi\| \\ &\le -(1-\theta)\sigma_{\min}(\Theta_1)\|\xi\|^2 \end{aligned} \tag{18}$$

for $\theta \in (0, 1)$. Let:

$$\mu = \frac{\sigma_{\max}(PB)\sup_{t\in[0,\infty)}\|\omega_{\mathrm{r}}\|}{\theta\sigma_{\min}(\Theta_1)} \tag{19}$$

Following the standard step provided in reference (Khalil and Grizzle, 2001), the ultimate bound of system can be given as:

$$\|\xi\| \le \|\xi\|_{\mathrm{b}} = \sqrt{\frac{\sigma_{\max}(\Xi_2)}{\sigma_{\min}(\Xi_1)}}\,\mu = \sqrt{\frac{\sigma_{\max}(\Xi_2)}{\sigma_{\min}(\Xi_1)}}\,\frac{\sigma_{\max}(PB)\sup_{t\in[0,\infty)}\|\omega_{\mathrm{r}}\|}{\theta\sigma_{\min}(\Theta_1)} \tag{20}$$

Proof of Step 2

Consider the Lyapunov function by omitting the fourth term of equation (14):

$$V_2 = (k_p + ck_d)((1 - q_0)^2 + q_{\mathrm{e}}^{\mathrm{T}}q_{\mathrm{e}}) + \frac{1}{2}\omega_{\mathrm{e}}^{\mathrm{T}}\mathcal{J}_{\mathrm{mb}}\omega_{\mathrm{e}} + cq_{\mathrm{e}}^{\mathrm{T}}\mathcal{J}_{\mathrm{mb}}\omega_{\mathrm{e}} \tag{21}$$

The time derivative of V_2 can be obtained as:

$$\begin{aligned}\dot{V}_2 = {} & (k_p + ck_d)q_{\mathrm{e}}^{\mathrm{T}}\omega_{\mathrm{e}} + s^{\mathrm{T}}(\tau + g) - \omega_{\mathrm{e}}^{\mathrm{T}}(M \\ & + (C_{\mathrm{br}}\omega_{\mathrm{r}})^{\times}\mathcal{J}_{\mathrm{mb}} + \mathcal{J}_{\mathrm{mb}}(C_{\mathrm{br}}\omega_{\mathrm{r}})^{\times})\omega_{\mathrm{e}} \\ & + q_{\mathrm{e}}^{\mathrm{T}}K_4\omega_{\mathrm{e}} + \frac{1}{2}c\omega_{\mathrm{e}}^{\mathrm{T}}(q_{\mathrm{e}0}I + q_{\mathrm{e}}^{\times})\mathcal{J}_{\mathrm{mb}}\omega_{\mathrm{e}} \end{aligned} \tag{22}$$

where:

$$\begin{aligned} g = {} & -(C_{\mathrm{br}}\omega_{\mathrm{r}})^{\times}\mathcal{J}_{\mathrm{mb}}C_{\mathrm{br}}\omega_{\mathrm{r}} - (C_{\mathrm{br}}\omega_{\mathrm{r}})^{\times}H\vartheta + L\vartheta - MC_{\mathrm{br}}\omega_{\mathrm{r}} \\ & - \mathcal{J}_{\mathrm{mb}}C_{\mathrm{br}}\dot{\omega}_{\mathrm{r}} + d \\ K_4 = {} & c(\mathcal{J}_{\mathrm{mb}}C_{\mathrm{br}}\omega_{\mathrm{r}})^{\times} - c(C_{\mathrm{br}}\omega_{\mathrm{r}})^{\times}\mathcal{J}_{\mathrm{mb}} - c\mathcal{J}_{\mathrm{mb}}(C_{\mathrm{br}}\omega_{\mathrm{r}})^{\times} \\ & - cM + c(H\vartheta)^{\times} \end{aligned}$$

Substituting the control law (11) into (22), then:

$$\dot{V}_2 \le -\varsigma^{\mathrm{T}}\Theta_2\varsigma - \sum_{i=1}^{3}\left(\lambda - \sup_{t\in[0,\infty)}|g_i(t)|\right)|s_i| \tag{23}$$

where

$$\varsigma = (\|q_{\mathrm{e}}\| \ \ \|\omega_{\mathrm{e}}\|)^{\mathrm{T}}$$

$$\Theta_2 = \begin{pmatrix} ck_p & -\frac{1}{2}\sigma_{\max}(K_4) \\ -\frac{1}{2}\sigma_{\max}(K_4) & k_d + \sigma_{\min}(M) - \frac{1}{2}c\sigma_{\max}(\mathcal{J}_{\mathrm{mb}}) \end{pmatrix}$$

If Θ_2 is positive definite for proper parameters and $\lambda > \sup_{t\in[0,\infty)}|g_i(t)|$, $i = 1, 2, 3$, it follows by $\lim_{t\to\infty}\varsigma = 0$ via the Lyapunov theorem, that is, $\lim_{t\to\infty}\varsigma = 0$, that is, $\lim_{t\to\infty}q_{\mathrm{e}}(t) = 0$ and $\lim_{t\to\infty}\omega_{\mathrm{e}}(t) = 0$.

If Θ_2 is semi-positive definite for some parameters and $\lambda > \sup_{t\in[0,\infty)}|g_i(t)|$, $i = 1, 2, 3$, then:

$$\dot{V}_2 \le -\mu\sum_{i=1}^{3}|s_i| \le -\mu\|s\| \tag{24}$$

where $\mu = \min_{i=1,2,3}(\lambda - \sup_{t\in[0,\infty)}|g_i(t)|)$. Now integrating both sides of equation (24), we have:

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$$V_2(t) \leq V_2(0) - \mu \int_0^t \|s\| dt \tag{25}$$

which can be further written as:

$$\|s\|_{L_1} = \mu \lim_{t\to\infty} \int_0^t \|s\| dt \leq V_2(0) - V_2(t) \leq V_2(0) \tag{26}$$

Hence, $\|s\| \in L_1$, and further, $\|s\| \in L_\infty$. From the kinematic and dynamic equations, it can be concluded that $\|\dot{s}\| \in L_\infty$. Hence, by Lemma 2, it follows that $\lim_{t\to\infty} \|s(t)\| = 0$.

Because $\lim_{t\to\infty} \|s(t)\| = 0$, there exists some finite time T such that:

$$\lim_{t\to\infty} \int_0^t \|s\|^2 dt - \lim_{t\to\infty} \int_0^t \|s\| dt = \lim_{t\to\infty} \int_0^t \|s\|(\|s\| - 1) dt \leq 0, \ \forall t > T \tag{27}$$

that is, $\|s\| \in L_2$.

To get the conclusion $\lim_{t\to\infty} \|q_e(t)\| = 0$ and $\lim_{t\to\infty} \|\omega_e(t)\| = 0$, the following positive function is considered:

$$V_3 = (k_p + ck_d)\left((1 - q_0)^2 + q_e^T q_e\right) \tag{28}$$

Its time derivative can be bounded as:

$$\dot{V}_3 = q_e^T \omega_e = q_e^T (s - cq_e) \leq -c\|q_e\|^2 + \|q_e\|\|s\| \tag{29}$$

By the previous similar analysis, it can be concluded that $\|q_e\| \in L_2 \cap L_\infty$ and $\|\dot{q}_e\| \in L_\infty$. Further using Lemma 2 again, we get $\lim_{t\to\infty} \|q_e(t)\| = 0$. Combining it with $\lim_{t\to\infty} \|s(t)\| = 0$, it can be shown that $\lim_{t\to\infty} \|\omega_e(t)\| = 0$.

Remark 1

To ensure the stability of the control system, the selection of control parameters should be taken into account such that Θ_1 and Θ_2 are positive definite, hence, c must be sufficiently small and $\lambda > \sup_{t\in[0,\infty)} |g_i(t)|, i = 1, 2, 3$ and $\lambda \geq \sup_{t\in[0,\infty)} |f_i(t)|, i = 1, 2, 3$.

In view of the positive definite Θ_1, the following two conditions should be satisfied:

$$ck_p\left(k_d + \sigma_{\min}(M) - \frac{1}{2}c\sigma_{\max}(\mathcal{J}_{mb})\right) > \frac{1}{4}\sigma^2_{\max}(K_1) \tag{30}$$

and the positive definite of $U - VW^{-1}V^T$. Because $U - VW^{-1}V^T$ satisfies

$$\chi^T(U - VW^{-1}V^T)\chi \geq \left(\sigma_{\min}(U) - \frac{\sigma^2_{\max}(V)}{\sigma_{\min}(W)}\right)\|\chi\|^2, \ \forall \chi \in R^n$$

if

$$\sigma_{\min}(U)\sigma_{\min}(W) > \sigma^2_{\max}(V) \tag{31}$$

the matrix $U - VW^{-1}V^T$ is positive definite.

Considering Θ_2 is positive definite, the following inequality is satisfied:

$$ck_p\left(k_d + \sigma_{\min}(M) - \frac{1}{2}c\sigma_{\max}(\mathcal{J}_{mb})\right) > \frac{1}{4}\sigma^2_{\max}(K_4) \tag{32}$$

There must exist sufficiently large k_p and k_d, as well as sufficiently small c, to make (30)-(32) satisfied.

Remark 2

To avoid the chattering phenomenon due to the imperfect implementation of the sign function in the control law (11), the function tanh (s/ε) is a simple choice to replace the sign function, where ε is a small positive parameter, and tanh (x) is the hyperbolic tangent function, which is defined as:

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

Simulation results

To demonstrate the effectiveness of the proposed control law (11), numerical simulations are performed and presented in this section. The main parameters are as follows:

$$\mathcal{J} = \begin{pmatrix} 800 & 12 & 5 \\ 12 & 400 & 1.5 \\ 5 & 1.5 & 600 \end{pmatrix} \tag{33}$$

$$\delta = \begin{pmatrix} 10 & 0.5 & 0.2 \\ 0.5 & 2 & 0 \\ 0.1 & 10.9 & 0.8 \\ 1 & 0.5 & 0.5 \end{pmatrix} \tag{34}$$

where $\mathcal{J}$ is presented in Kg·m^2 and δ is presented in Kg$^{1/2}$·m. Four elastic modes have been taken into account in the model used for simulating with the natural frequencies (in rad/s):

$$\omega_n = [1.9 \ \ 4.1 \ \ 5.8 \ \ 6]^T \tag{35}$$

dampings:

$$\zeta = [0.05 \ \ 0.09 \ \ 0.16 \ \ 0.25]^T \tag{36}$$

the modal variable initial values:

$$\eta_i = \dot{\eta}_i = 0, \ i = 1, \cdots, 4.$$

The initial angular velocity is [0.2 −0.3 0.2] rad/s. The initial attitude is described by the quaternion [0.7071 0.4082 0.4082 0.4082].

In addition, simulation was done corresponding to the following disturbance torque:

$$d = \sin(t)(0.4 \ \ -0.3 \ \ 0.7)^T \text{Nm} \tag{37}$$

and the desired tracking trajectory is produced by the equation (3) with input:

$$\omega_d = (0.1\cos(0.5t) - 0.1\sin(0.4t) \ \ -0.1\cos(0.3t))^T \text{ rad/s} \tag{38}$$

and

$$q_{d0} = [0.8771 \ \ 0.8771 \ \ 0.1754 \ \ 0.4384]$$

Figure 1 Attitude tracking errors

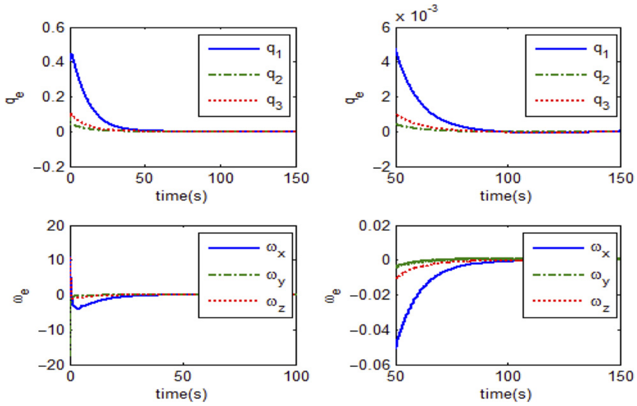


Figure 2 Response of the modal coordinates

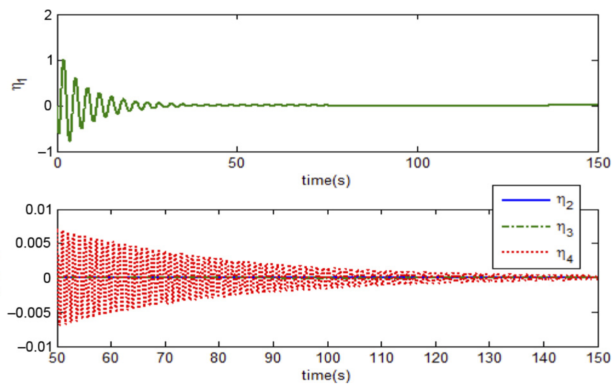
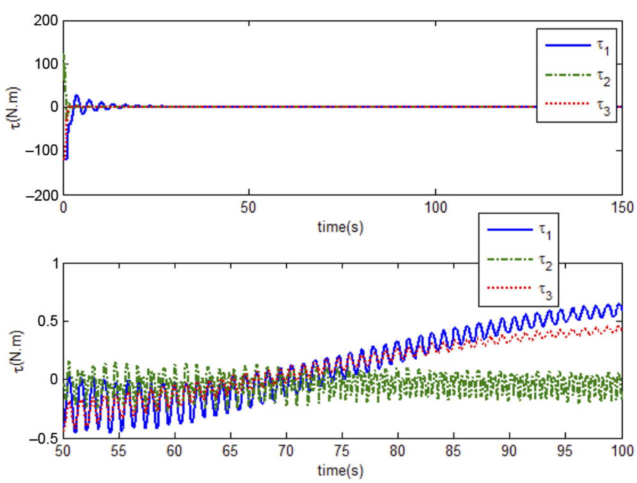


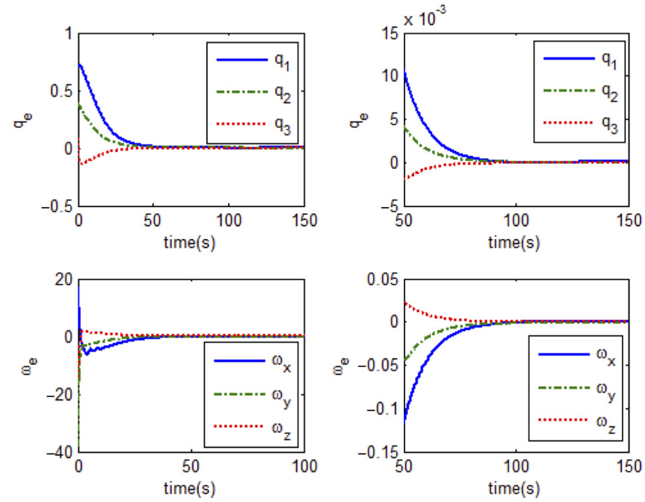
Figure 3 Control torque input



The parameters for control law (11) are chosen as $c = 0.13$, $k_p = 220$, $k_d = 2300$, $\varepsilon = 0.0025$ and $\lambda_i = 200$, $i = 1, 2, 3$.

The simulation results are shown in Figures 1-3. From Figure 1, it is easily seen that the tracking errors of attitude and angular velocity are well convergent, and they will converge to zero as time goes. Form time history of the modal coordinates of flexible appendage presented in Figure 2, the modal variables are limited to a steady level. The control torques are shown in Figure 3, which indicates that the tracking problem is effectively settled by the control law (11). Because the reference angular velocity is in the form of trigonometric function, the corresponding applied control torques from Figure 3 approximate the harmonic curves.

Figure 4 Attitude tracking errors in a different scenario



To show that the controller can stabilize the attitude tracking problem from any initial state, another simulation is conducted in a different scenario and is described as followed. The initial angular velocity, the initial quaternion and the desired angular velocity are changed to $[0.3\ \ -0.65\ \ -0.25]$, $[0.3536\ \ 0.5000\ \ 0.7500\ \ 0.2500]$ and $(0.1\cos(0.7t)\ -\ 0.1\sin(0.4t)\ -\ 0.1\cos(0.5t))^{\mathrm{T}}$rad/s, respectively. The other parameters were kept the same as in the preceding case. For brevity's sake, only the attitude and the angular velocity tracking errors of the simulation are presented in Figure 4. Compared with the previous simulation results in Figure 1, it showed that the control objective is still achieved, so the stability of the tracking problem can be guaranteed with the controller from any initial state to a certain extent.

Conclusion

In this research, a novel robust attitude tracking controller for a flexible spacecraft is proposed. The controller is the one with the structure of a PD term plus a switching function about a sliding variable, and the relative attitude is described by quaternion. The globally asymptotic stability of the controller in the presence of model uncertainties and external disturbances is proven rigorously through a two-step proof technique. Numerical simulations are carried out to support the analysis of the control law presented. The results demonstrate that fine convergence of the attitude and angular velocity error and low-level vibration of flexible appendages is obtained by the proposed controllers.

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About the authors

Dong Ye received BS, MS and PhD from Harbin Institute of Technology. Since 2013, he has been working as a Lecturer at Research Centre of Satellite Technology, Harbin Institute of Technology. His research interests include hardware-in-loop simulation technique, spacecraft dynamics and control technique. Dong Ye is the corresponding author and can be contacted at: yed@hit.edu.cn

Zhaowei Sun received the masters degree in general mechanics from Beijing Institute of Technology, China, in 1988 and PhD in spacecraft design from Harbin Institute of Technology, Harbin, China, in 2002. Since 1997, he has been working as a Professor at Harbin Institute of Technology, Harbin, China. His research interests include spacecraft system design and simulation technique, spacecraft dynamics and control technique.